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$$h = \frac{V}{\pi r^2}, \quad r = \sqrt{\frac{V}{\pi h}}$$

$$\Rightarrow SA = 2\pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right)$$

$$= 2\pi r^2 + \frac{2V}{r}$$

$$\frac{d}{dr} SA = 4\pi r - \frac{2V}{r^2}$$

$$\frac{d}{dr} SA = 0$$

$$\Rightarrow 4\pi r - \frac{2V}{r^2} = 0$$

$$4\pi r^3 - 2V = 0$$

$$r^3 = \frac{2V}{4\pi}$$

$$= \frac{V}{2\pi}$$

$$r = \sqrt[3]{\frac{V}{2\pi}}$$

Domain of r: $0 < r < \infty$

Discontinuities: $r = 0$

$$\lim_{r \to 0^+} SA = 0 + \frac{2V}{0^+}$$

$$= \infty$$

End points:

$$\lim_{r \to \infty} SA = \infty$$

$\Rightarrow r = \sqrt[3]{\frac{V}{2\pi}}$ is the minimum point as it is the only critical point.

$$h = \frac{V}{\pi\left(\frac{V}{2\pi}\right)^{\frac{2}{3}}} \qquad \therefore \frac{r}{h} = \left(\frac{\frac{V}{2\pi}}{\frac{4V}{\pi}}\right)^{\frac{1}{3}}$$

$$= \frac{2^{\frac{2}{3}} V^{\frac{1}{3}}}{\pi^{1/3}} \qquad = \sqrt[3]{\frac{1}{8}}$$

$$= \sqrt[3]{\frac{4V}{\pi}} \qquad = \frac{1}{2}$$